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POVRŠINA, MOMENTI I TEŽIŠTE ODSJEČKA ELIPSE

<u>Osnove</u>

$(x/a)^2 + (y/b)^2 = 1$

$\mathrm{tg}\phi = y/x$, za točku $T(x, y)$

$x = a\cdot\cos t$

$y = b\cdot\sin t$

$\mathrm{tg}\phi = b\cdot\sin t/(a\cdot\cos t)$

$\mathrm{tg}\phi = b/a\cdot\mathrm{tg}t$

$\mathrm{tg}t = a/b\cdot\mathrm{tg}\phi$

$\rho_1 = (x_1^2 + y_1^2)^{0.5} = ab/(b^2\cos^2\phi_1 + a^2\sin^2\phi_1)^{0.5}$

$\rho_2 = (x_2^2 + y_2^2)^{0.5} = ab/(b^2\cos^2\phi_2 + a^2\sin^2\phi_2)^{0.5}$

$\cos t = 1/(1+\mathrm{tg}^2 t)^{0.5} = 1/(1 + a^2/b^2\cdot\mathrm{tg}^2\phi)^{0.5}$

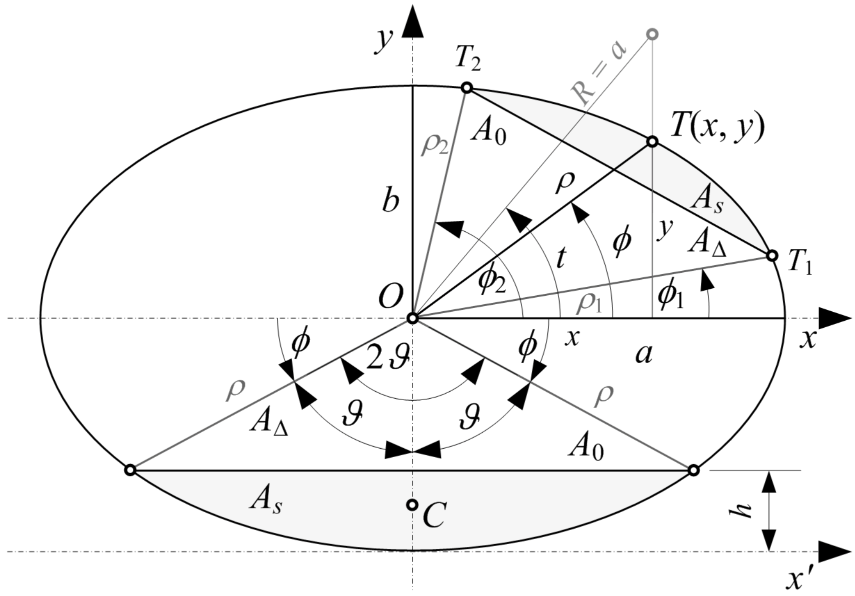
$\cos t = 1/(1+a^2\sin^2\phi/(b^2\cdot\cos^2\phi))^{0.5}$

$\cos t = b\cdot\cos\phi/(b^2\cos^2\phi + a^2\sin^2\phi)^{0.5}$

$\sin t = \mathrm{tg}t/(1+\mathrm{tg}^2 t)^{0.5} = a/b\cdot\mathrm{tg}\phi/(1+a^2/b^2\cdot\mathrm{tg}^2\phi)^{0.5}$

$\sin t = a\cdot\sin\phi/(b\cdot\cos\phi)/(1+a^2\sin^2\phi/(b^2\cdot\cos^2\phi))^{0.5}$

$\sin t = a\cdot\sin\phi/(b^2\cos^2\phi + a^2\sin^2\phi)^{0.5}$



<u>Odnos između središnjeg polarnog kuta ϕ i parametra t</u>

$$\boldsymbol{t = \mathrm{arctg}(a/b\cdot\mathrm{tg}\phi) + k\pi,\ k \in \mathbb{Z}}$$

<u>Površina odsječka elipse</u>

$A_S = A_0 - A_\Delta$

(Opći Greenov teorem)

$A_0 = 1/2\cdot|{}_{t1}\!\int^{t2}(xdy - ydx)|$

$dx = -a\cdot\sin t\cdot dt$

$dy = b\cdot\cos t\cdot dt$

$A_0 = 1/2\cdot|{}_{t1}\!\int^{t2}[a\cdot\cos t(b\cdot\cos t)\cdot dt$
$\quad - b\cdot\sin t(-a\cdot\sin t)\cdot dt]|$

$A_0 = ab/2\cdot|{}_{t1}\!\int^{t2}(\cos^2 t + \sin^2 t)dt|$

$A_0 = ab/2\cdot|{}_{t1}\!\int^{t2}dt|$

$\boldsymbol{A_0 = ab|t_2 - t_1|/2}$

$A_\Delta = \rho_1\cdot\rho_2\cdot\sin(\phi_2 - \phi_1)/2$

$\boldsymbol{A_\Delta = 1/2\cdot ab|1/(a^2\cos^2\phi_1 + b^2\sin^2\phi_1)^{0.5}}$
$\boldsymbol{\cdot 1/(a^2\cos^2\phi_2 + b^2\sin^2\phi_2)^{0.5}\cdot\sin(\phi_2 - \phi_1)|}$

$\boldsymbol{A_S = ab/2\cdot|\mathrm{arctg}(a/b\cdot\mathrm{tg}\phi_2) + k_2\pi}$
$\boldsymbol{- \mathrm{arctg}(a/b\cdot\mathrm{tg}\phi_1) - k_1\pi|}$
$\boldsymbol{-1/2\cdot|1/(a^2\cos^2\phi_1 + b^2\sin^2\phi_1)^{0.5}}$
$\boldsymbol{\cdot 1/(a^2\cos^2\phi_2 + b^2\sin^2\phi_2)^{0.5}\cdot\sin(\phi_2 - \phi_1)|}$

<u>Momenti općeg odsječka elipse</u>

$M_{s,x} = M_{0,x} - M_{\Delta,x} = M_{0,x} - A_\Delta y_{\Delta,C}$

(Opći Greenov teorem)

$M_{0,x} = 1/3\cdot{}_{t1}\!\int^{t2}y\cdot(xdy - ydx)$

$M_{0,x} = ab/3\cdot{}_{t1}\!\int^{t2}b\cdot\sin t\cdot(\cos^2 t + \sin^2 t)dt$

$M_{0,x} = ab^2/3\cdot{}_{t1}\!\int^{t2}\sin t\cdot dt$

$\boldsymbol{M_{0,x} = -ab^2/3\cdot(\cos t_2 - \cos t_1)}$

$M_{0,x} = -ab^2/3\cdot(1/(1+a^2/b^2\cdot\mathrm{tg}^2\phi)^{0.5}$
$\quad -1/(1+a^2/b^2\cdot\mathrm{tg}^2\phi)^{0.5})$

$\boldsymbol{y_{\Delta,C} = 1/3\cdot b\cdot(\sin(t_1)+\sin(t_2))}$

$y_{\Delta,C} = 1/3\cdot b\cdot(a/b\cdot\mathrm{tg}\phi_1/(1+a^2/b^2\cdot\mathrm{tg}^2\phi_1)^{0.5}$
$\quad +a/b\cdot\mathrm{tg}\phi_2/(1+a^2/b^2\cdot\mathrm{tg}^2\phi_2)^{0.5})$

$\boldsymbol{M_{S,x} = - ab^3/3[\cos\phi_2/(b^2\cos^2\phi_2+a^2\sin^2\phi_2)^{0.5}}$
$\boldsymbol{- \cos\phi_1/(b^2\sin^2\phi_1+a^2\sin^2\phi_1)^{0.5}]}$
$\boldsymbol{- abA_\Delta/3[\sin\phi_1/(b^2\cos^2\phi_1+a^2\sin^2\phi_1)^{0.5}}$
$\boldsymbol{+ \sin\phi_2/(b^2\sin^2\phi_2+a^2\sin^2\phi_2)^{0.5}]}$

$M_{S,y} = M_{0,y} - M_{\Delta,y} = M_{0,y} - A_\Delta x_{\Delta,C}$

(Opći Greenov teorem)

$M_{0,y} = 1/3{}_{t1}\!\int^{t2}x\cdot(xdy - ydx)$

$M_{0,y} = ab/3\cdot{}_{t1}\!\int^{t2}a\cdot\cos t\cdot(\cos^2 t + \sin^2 t)dt$

$M_{0,y} = a^2b/3\cdot{}_{t1}\!\int^{t2}\cos t\cdot dt$

$\boldsymbol{M_{0,y} = a^2b/3\cdot(\sin t_2 - \sin t_1)}$

$M_{0,y} = a^2b/3\cdot[a/b\cdot\mathrm{tg}\phi_2/(1+a^2/b^2\cdot\mathrm{tg}^2\phi_2)^{0.5}$
$\quad - a/b\cdot\mathrm{tg}\phi_1/(1+a^2/b^2\cdot\mathrm{tg}^2\phi_1)^{0.5}]$

$\boldsymbol{x_{\Delta,C} = 1/3\cdot a\cdot(\cos(t_1)+\cos(t_2))}$

$x_{\Delta,C} = 1/3\cdot a\cdot(1/(1+a^2/b^2\cdot\mathrm{tg}^2\phi_1)^{0.5}$
$\quad +1/(1+a^2/b^2\cdot\mathrm{tg}^2\phi_2)^{0.5})$

$\boldsymbol{M_{S,y} = a^3b/3[\sin\phi_2/(b^2\cos^2\phi_2+a^2\sin^2\phi_2)^{0.5}}$
$\boldsymbol{- \sin\phi_1/(b^2\sin^2\phi_1+a^2\sin^2\phi_1)^{0.5}]}$
$\boldsymbol{- abA_\Delta/3[\cos\phi_1/(b^2\cos^2\phi_1+a^2\sin^2\phi_1)^{0.5}}$
$\boldsymbol{+ \cos\phi_2/(b^2\sin^2\phi_2+a^2\sin^2\phi_2)^{0.5}]}$

Primjer: **<u>Horizontalni odsječak elipse</u>**

Zadana je visina h tekućine u cilindričnom tanku poprečnog presjeka u obliku elipse.
Odsječak volumena u tanku ima horizontalno simetričan oblik, širine kuta: 2ϑ.
Potrebno je odrediti površinu A_S i momente odsječka M_S.

$x_{1,2} = \mp x \quad y_{1,2} = h - b = konst.$ $\qquad \vartheta_1 = 3/2\pi - \vartheta,\ \vartheta_2 = 3/2\pi + \vartheta$ $\qquad \vartheta = \pi/2 - \phi$

$\phi_1 = \pi + \phi,\ \phi_2 = 2\pi - \phi,$ $\qquad t_1 = \pi + t,\ t_2 = 2\pi - t$

$\mathrm{tg}(\phi) = \mathrm{tg}(\pi/2 - \vartheta) = b(b - h)/[a(2bh - h^2)^{0.5}]$ $\qquad \mathrm{tg}(t) = (b - h)/(2bh - h^2)^{0.5}$

$A_S = A_0 - A_\Delta$

$A_\Delta = a/b(b - h)(2bh - h^2)^{0.5}$

$A_0 = 1/2\cdot ab\cdot|\pi - 2\mathrm{arctg}((b - h)/(2bh - h^2)^{0.5})|$ $\qquad M_{S,y} = 0$

$M_{S,x} = M_{0,x} - M_{\Delta,x}$

$M_{S,x} = 2/3\cdot ab\cdot(A_\Delta\cdot\tan(\phi) - b^2)\cdot(b^2\cdot\cos(\phi) - a^2\cdot\sin(\phi))^{0.5})$

$M_{S,x} = 2a/(3b)\cdot(2bh - h^2)^{1.5}$

$\boldsymbol{A_S = ab/2\cdot|\pi - 2\cdot\mathrm{arctg}((b - h)/(2bh - h^2)^{0.5})| - a/b\cdot(b - h)\cdot(2bh - h^2)^{0.5}}$

$\boldsymbol{M_{S,x} = 2a/(3b)\cdot(2bh - h^2)^{1.5}}$